

Brief communication

“Models for the exceedances of high thresholds over the precipitation daily totals in Athens, Greece”

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Abstract. Extreme precipitation events have significant environmental consequences because they may cause considerable damages to urban as well as rural areas. The aim of this work is to construct a threshold model which will describe the exceedances over a threshold for the daily precipitation totals over Athens, Greece. The data used are daily precipitation totals recorded at the National Observatory of Athens, for a 115-year period (1891–2005).

The generalized Pareto distribution is considered as the proper distribution for the study of the exceedances. The threshold of $u=15.8$ mm (10% upper limit) is used for the construction of the optimal return level function.

1 Introduction

Precipitation is the most changeable, of all climatic parameters, in time and in space. There is a need to analyze precipitation variability, for assessing possible adverse implications concerning precipitation rate, soil erosion, floods and droughts. The frequency of heavy precipitation events has increased over most land areas, consistent with warming and observed increases of atmospheric water vapour (IPCC, 2007).

With respect to the precipitation variability in Greece a lot of studies have been carried out (Repapis, 1986; Katsoulis and Kambezidis, 1989; Amanatidis et al., 1993; Metaxas et al., 1999; Houssos and Bartzokas, 2006; Pnevmatikos and Katsoulis, 2006; Feidas et al., 2007). Nastos and Zerefos (2008) in a more recent study, found that increased variance and scale parameter of the fitted Gamma distributions

on the precipitation data sets of Greece appears in the western and southern-eastern regions, especially during the last decade 1991–2000, indicating the incidence of extreme daily precipitation.

As far as the temporal variability of precipitation and extreme events in the Athens province is concerned, a lot of research has been fulfilled by using precipitation data sets recorded in the National Observatory of Athens (NOA) (Katsoulis and Kambezidis, 1989; Amanatidis et al., 1993; Paliatsos et al., 2005; Nastos and Zerefos, 2007). Changes in the frequency of precipitation events, or in the intensity of precipitation, or both, assign the variability of the total precipitation (Nastos and Zerefos, 2007). However, extreme precipitation is difficult to reproduce, especially for the intensities and patterns of extreme events, which are heavily affected by the local scale (IPCC, 2001). For this reason, it is necessary to study these events by analyzing long time series of observations.

The goal of this study is the construction of a threshold model that describes the exceedances over a threshold for the daily precipitation totals over Athens area. The main difficulty in the statistical analysis is the dependence among the daily observations which make the statistical theorems non valid because they assume independent observations. However, this dependence is short-term in the sense that daily observations separated by sufficiently large interval of time are considered approximately independent. Thus, with practical application in mind, it is usual to assume a condition that the events $X_i > u$ and $X_j > u$ are approximately independent, provided that the threshold u is high enough, and the time points i and j are far enough apart. It becomes apparent that in order to construct a valid return period function for daily observations, we must firstly construct a proper exceedance of a high threshold model.



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Table 1. Main statistical features of the data for different values of the threshold.

	Threshold u (mm)	n_u (exceedances)	n_c (clusters)	mean (mm)	sd (mm)	skewness
20%	9.1	1605	1215	19.95	13.77	4.15
15%	11.8	1215	981	23.02	14.54	4.11
10%	15.8	814	692	27.69	15.7	4.0
5%	43.9	79	76	63.67	26.1	2.61
Total	–			5.79	9.58	4.86

2 Data and methodology

In this study, the daily precipitation totals recorded at NOA for a 115-year period (1891–2005) were used. This time series of NOA (Longitude: 23°43' E, Latitude: 37°58' N, Altitude: 107 m a.m.s.l.) is the longest available record of daily precipitation totals in Greece. Its analysis aims at the determination of the appropriate threshold for extreme precipitation events.

The return period function is studied with the help of the generalized extreme value (GEV) and the Pareto distribution. The use of these distributions for the analysis of extreme events is justified from some classical theorems from Extreme Value Theory (Gnedenko, 1943; Gumbel, 1958; Pickands, 1975).

However, the daily precipitations are not independent events. To overcome the dependence problem Davinson and Smith (1990) proposed the following procedure:

1. We must define the cluster of excesses. Two consecutive clusters are separated by r-consecutive days of no excesses.
2. Within each cluster we identify the maximum excess.
3. We assume cluster maxima to be independent, with distribution given by the generalized Pareto distribution.
4. The level of exceedance x_m that is the value exceeded on average once every m -observations is given from the relation

$$x_m = u + \frac{\sigma_0}{\xi} [(m P(X > u) \theta)^{\xi} - 1], \quad (1)$$

where the parameter θ is called extremal index and approximately is

$$\theta = \frac{1}{\text{mean cluster size}}.$$

3 Results and discussion

The choice of r is an important task. Small r may leads to dependence of the excesses while large r may leads to loss of information. Since the phenomenon of precipitation has short memory, in the sense that the dependence of two precipitation events decays rapidly over time, in our study we consider $r=4$ as the smallest number of days which determine independent precipitation events. Thus the above procedure provides us with a sample of independent exceedances. From Eq. (1) above the influence of the choice of the threshold is obvious. Therefore we select the 80%, 85%, 90% and 95% daily precipitation total over the total time series. The parameter ξ is independent of u and thus it must remain unaltered despite the choice of the threshold u . The parameters are estimated with the maximum likelihood method.

Table 1 shows the main statistical features for various values of thresholds which are useful for the estimation procedure.

The probability of an exceedance over u $P(X > u)$ and the parameters θ are estimated by $P(X > u) = \frac{n_u}{n}$ and $\hat{\theta} = \frac{n_c}{n}$, respectively. Table 2 summarizes the estimators of the various parameters involved.

The return level function which assigns the daily precipitation level x_m that is exceeded on average once every m daily observations, is given from Eq. (1). In order to construct the m -year return level function, Coles (2001) proposes the following modification of (1)

$$x_m = R_1(m) = u + \frac{\sigma_0}{\xi} [(360m P(X > u) \theta)^{\xi} - 1] \quad (2a)$$

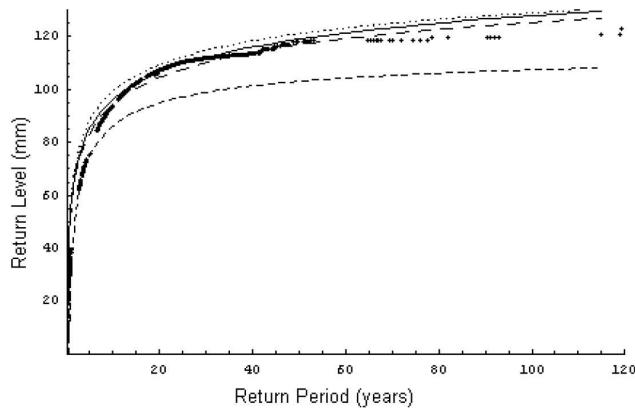
However intuitively it is not appropriate to consider 360 observations per year than the average observations per year. In our 115-year period the average annual precipitation days is 70. Thus instead of Eq. (2a) we adopt the m -year return level function

$$x_m^* = R_2(m) = u + \frac{\sigma_0}{\xi} [(70m P(X > u) \theta)^{\xi} - 1] \quad (2b)$$

The return level plots for the 80%, 85%, 90%, and 95% highest value, are shown in Fig. 1. We observe that the return level based on the 90% highest value describes the data

Table 2. Parameter estimation for different values of the threshold.

	threshold (mm)	$\hat{\theta}$	$P(X > u)$	$\hat{\xi}$	$\hat{\sigma}_0$
20%	9.1	0.75	0.2	-0.09	23.05
15%	11.8	0.81	0.141	-0.102	23.42
10%	15.8	0.85	0.096	-0.147	25.89
5%	43.9	0.96	0.010	-0.423	32.31

**Fig. 1.** Return level (mm) for the 80% (solid line), 85% (dash line), 90% (dense dash line) and 95% (dot line) highest value. (+): corresponds to the observed values.

better compared to the 80%, 85%, and 95% highest value. In conclusion, the m-day return level function which fits best to the data is:

$$x_m = 15.8 - 191.922 \left(\frac{1.4453}{m^{0.147}} - 1 \right) \quad (3a)$$

while for the m-year return level function the proposed function is:

$$x_m^* = 15.8 - 191.922 \left(\frac{0.77402}{m^{0.147}} - 1 \right) \quad (3b)$$

The choice of the optimal threshold for producing the best return level functions 3(a, b) with respect to the precipitations causing flood phenomena is based on a standard methodology as it appears in relevant papers (Coles, 2001; Davison and Smith, 1990). In a more recent paper, Pawlowsky-Glahn et al. (2005) using the same methodology concluded in similar results. More specifically, in order to find a reference threshold, they looked for a value such that the mean excess function assumed linear. In this way, they selected the reference $u=45$ mm in order to predict the return period with the highest accuracy.

The threshold u does not have the meaning of a threshold of extreme events but it is an essential term in the Eq. (2a, b) in order to construct the return level functions. The choice of this reference threshold is based upon the assumption that the mean excess function is linear. From this point of view, the meteorological importance of the threshold $u=15.8$ mm (10% upper limit) in the wider area of Athens is that it allows us to predict extreme precipitation events (associated more likely to flood phenomena), with the help of the return level functions. This approach is important because according to Koukis and Koutsoyiannis (1997), flooding in Athens is probably the most severe among hydrometeorological hazards in Greece. At least 179 lives were lost due to floods in Athens since 1896, with the most human loss flood events appeared on 14 November 1896 (61 deaths), 5–6 November 1961 (40 deaths), and 2 November 1977 (38 deaths), (Nicolaidou and Hadjichristou, 1995).

4 Conclusions

The aim of this paper is to construct a threshold model for daily precipitation totals. The choice of the reference threshold is supported by many validation tools. The return level function 3(a, b) based on the 90% highest value describes the data better compared to the 80%, 85%, and 95% highest value. This function is very important for the hydrology because it allows us to compute the value x_m exceeded on average once every m daily observations. Based on the m-observation return level function the m-year return level function is constructed which gives the value x_m exceeded on average once every m years. As a final comment we can say that the 10% upper limit ($u=15.8$ mm) for the time series under consideration provides us with the optimal excess as well as with the optimal return period function. However further studies must be made in order to examine if the 90% highest value is the optimal threshold for the daily precipitation time series from other regions.

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